Technische Universität München
Institut für Informatik
Theoretical Computer Science

WS 2018/2019

Fundamental Algorithms 1

Exercises

Exercise 1

Prove (by induction over n) that $\frac{1}{3}n^2 + 5n + 30 \in O(n^2)$ for all $n \in \mathbb{N}^+$.

Exercise 2

(a) Compare the growth of the following functions using the o-, O-, and Θ-notation:

1. $n \ln n$
2. n^l for all $l \in \mathbb{N}$
3. 2^n

Hint: use L'Hôpital's rule!

(b) Prove the following growth characterizations:

$$1)\ \sum_{i=1}^{n} \frac{1}{i} \in \Theta(\ln n) \qquad\qquad 2)\ \ln(n!) \in \Theta(n \ln n)$$

Hint: Try to prove $n^{\frac{n}{2}} \leq n! \leq n^n$ first!

Exercise 3

Let $l(x)$ be the number of bits of the representation of x in the binary system. Prove:

$$\sum_{i=1}^{n} l(i) \in \Theta(n \ln n)$$

Exercise 4

Prove that $\widehat{\Theta} = \{(f, g) \mid f \in \Theta(g)\}$ defines an equivalence relation on the set of functions $\{f \mid f : \mathbb{N} \to \mathbb{R}\}$.

Homework

Study the following basic algorithms for sorting:

InsertionSort: i.e., sort a data set by successively inserting individual items into a sorted list.

MergeSort: i.e., splitting a list into two halves, sorting the halves individually, and merging the sorted sublists → in particular, study the **Merge** algorithm for combining two sorted lists into one.

You should understand how each algorithm proceeds to sort a given list of items.